

Mathematics

Proportional Reasoning

Student Handbook



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The **Connected Learning Initiative (CLix)** is a technology enabled initiative at scale for high school students. The initiative was seeded by Tata Trusts, Mumbai and is led by Tata Institute of Social Sciences, Mumbai and Massachusetts Institute of Technology, Cambridge, MA USA. CLix offers a scalable and sustainable model of open education, to meet the educational needs of students and teachers. The initiative has won UNESCO's prestigious 2017 King Hamad Bin Isa Al-Khalifa Prize, for the Use of Information and Communication Technology (ICT) in the field of Education.

CLix incorporates thoughtful pedagogical design and leverages contemporary technology and online capabilities. Resources for students are in the areas of Mathematics, Sciences, Communicative English and Digital Literacy, designed to be interactive, foster collaboration and integrate values and 21st century skills. These are being offered to students of government secondary schools in Chhattisgarh, Mizoram, Rajasthan and Telangana in their regional languages and also released as Open Educational Resources (OERs).

Teacher Professional Development is available through professional communities of practice and the blended Post Graduate Certificate in Reflective Teaching with ICT. Through research and collaborations, CLix seeks to nurture a vibrant ecosystem of partnerships and innovation to improve schooling for underserved communities.

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Centre for Education Research & Practice – Jaipur, Department of Education, Mizoram University – Aizawl, Eklavya – Bhopal, Homi Bhabha Centre for Science Education, TIFR – Mumbai, National Institute of Advanced Studies – Bengaluru, State Council of Educational Research and Training (SCERT) of Telangana – Hyderabad, Tata Class Edge – Mumbai, Inter-University Centre for Astronomy and Astrophysics – Pune, Govt. of Chhattisgarh, Govt. of Mizoram, Govt. of Rajasthan and Govt. of Telangana.

Any questions, suggestions or queries may be sent to us at:
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Mathematics
Proportional Reasoning
Student Handbook

This handbook belongs to:

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Class:

Section:

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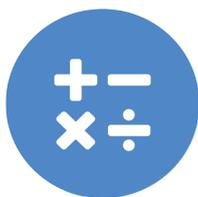
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Module Overview

About Proportional Reasoning Module

The proportional reasoning module is developed to facilitate students' shift from additive to multiplicative reasoning and to be able to appreciate the idea of scaling and recognizing scaling in situations involving the relationship between two or more quantities. The module is made for students of 8th and 9th classes. There are a total of 4 units in the Proportional Reasoning module, unit-1 is about the move from additive to multiplicative reasoning, unit-2 is about the multiplicative thinking, unit-3 is about Ratio and Proportion and unit 4 is about the application of proportional reasoning in different contexts. There are three major digital tools used in this module, food sharing tool, pattern task and ice-cube activity, each with several variations involved in it.

The modules have digital as well as hands-on activities and formative assessments form an integral part of this course. There are Pre and Post assessments at the start and end of each module on the digital platform. The course is prepared by conforming with the current State and NCERT syllabi and the activities designed in this course focus on developing thinking and reasoning abilities of the students. The Mathematics modules will be installed in the school computer labs using a server based model.

How to use this book ?

This book contains some of the activities of the module that are hands-on (involving classroom discussions) which are to be used in conjunction with other materials that are present on the CLix platform. These hands-on activities and worksheets help to elicit and consolidate learning of Geometrical shapes and should be done in a proper sequence along with the digital activities. Students may do the problems in the space provided in this workbook or in their notebooks and discuss with their teachers and peers.

The CLix platform is a digital platform that makes use of both the digital content and the workbook content. The platform has features like Notebook, Discussion and Gallery where students can give their responses, comments and upload their work respectively.

Unit 1: Additive to Multiplicative Thinking

Lesson 1.1 : Jamuni learns to share

*Please refer to this lesson on the **CLIX platform***

The five digital activities help in understanding equal share and fair distribution using fractions.

Lesson 1.2 : Let's compare and distribute

*Please refer to this lesson on the **CLIX platform***

The five digital activities help in understanding the concepts of proportional and additive reasoning.

Lesson 1.3 : Make the share equal across groups

*Please refer to this lesson on the **CLIX platform***

The four digital activities further clarify the concepts of proportional and additive reasoning.

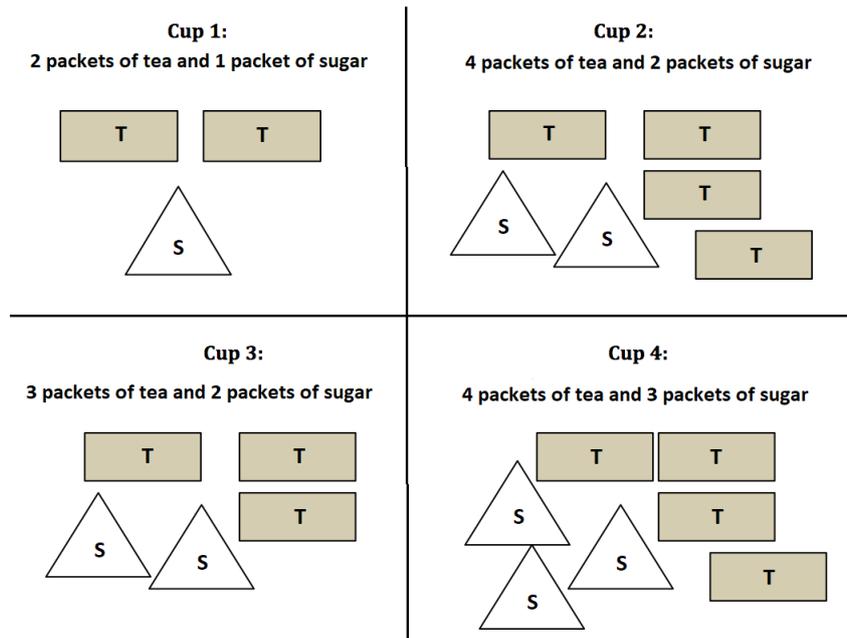
Unit 2: Multiplicative Thinking

Lesson 2.1 : Sweeter tea

Activity 1

In the image, each group of shapes represents a cup of tea. The rectangular shapes represent tea packets and the triangular shapes represent sugar packets.

Study each cup of tea carefully and find out which of the 4 cups of tea is most sweet.

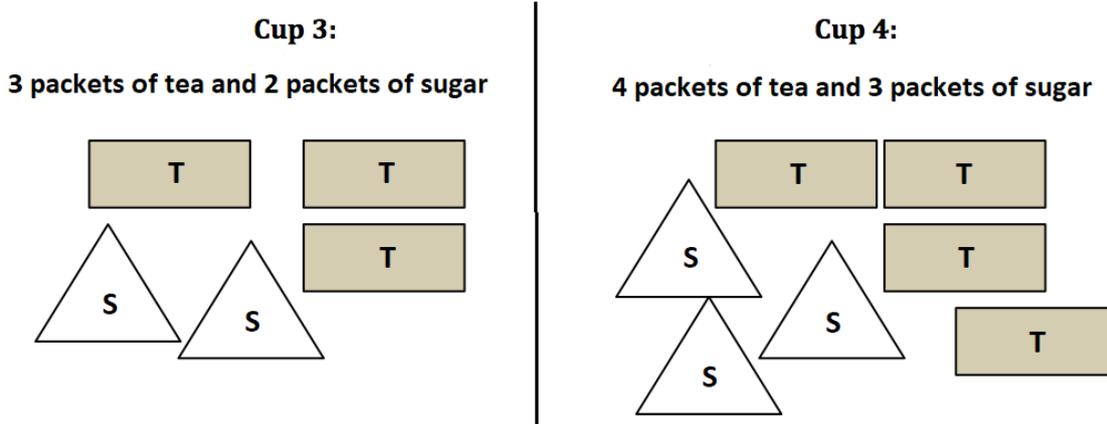


Activity 2

In the image, each group of shapes represents a cup of tea. The triangles represent sugar packets and the rectangles represent tea packets.

You are given 6 packets of tea. How many packets of sugar would you need to make a cup of tea that is exactly the same as:

- The one shown in Cup 3
- The one shown in Cup 4



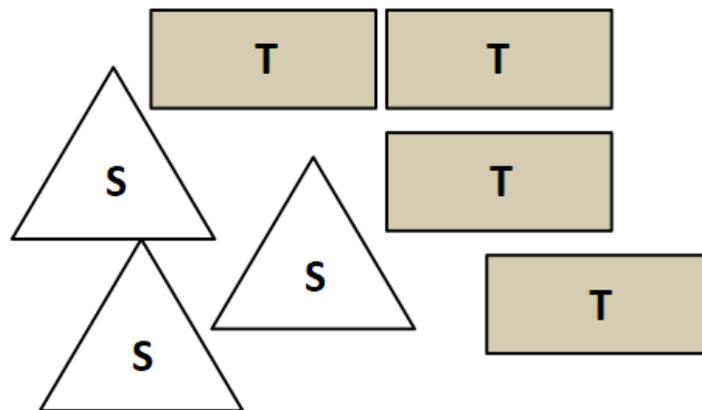
Activity 3 - Part 1

In the image, each group of shapes represents a cup of tea. The triangles represent sugar packets and the rectangles represent tea packets.

You have 15 packets of tea and 11 packets of sugar. How many packets of tea and sugar will you need to make 4 cups of tea that are as sweet as the one shown in Cup 4?

Cup 4:

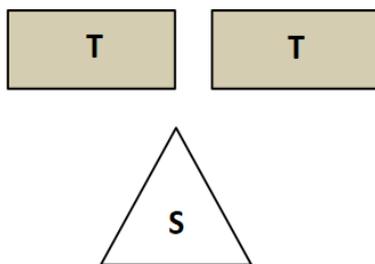
4 packets of tea and 3 packets of sugar



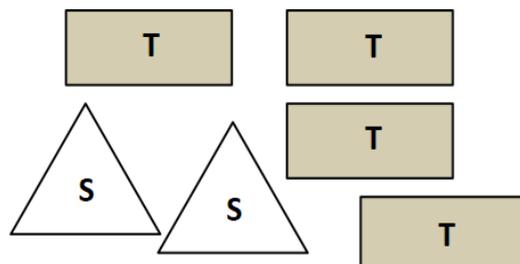
Activity 3 - Part 2

Study the following pictures of cups of tea. Each rectangle represents a packet of tea and each triangle represents a packet of sugar. Which cup has less sweeter tea?

Cup 1:
2 packets of tea and 1 packet of sugar



Cup 2:
4 packets of tea and 2 packets of sugar



Lesson 2.2 : Jamuni solves puzzles

Please refer to this lesson on the CLIX platform

The six digital activities help in building multiplicative thinking through scaling up and scaling down of patterns.

Lesson 2.3 : Jamuni goes to the bazaar

Activity 1

Jamuni and her friends are at an egg shop in the mela.

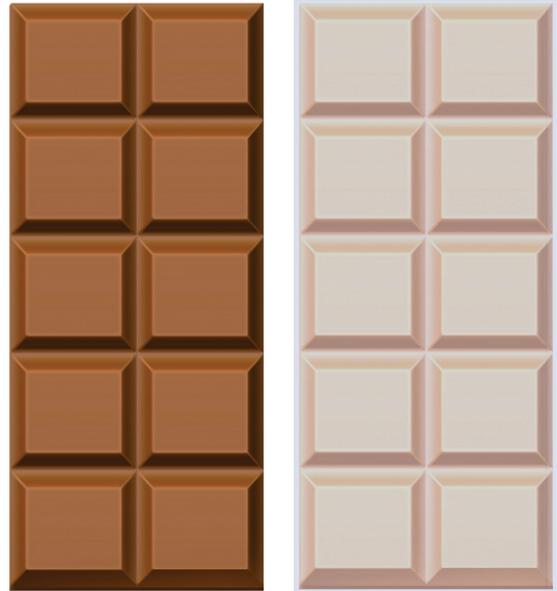
- They see a tray of eggs. The tray contains 12 eggs and costs Rs.36. Now if they want an egg tray that has one and half times more eggs than this tray, how much will they need to pay ?
- Shabhana is Jamuni's friend and she wants to buy two egg trays. She finds that there is a mix of both white and brown eggs in each tray. The first tray holds 12 eggs of which 4 are brown and 8 are white. The second tray holds 18 eggs. If the proportion of brown and white eggs is the same in both trays, how many eggs of each colour does the second tray have ?

	Tray 1	Tray 2
Total number of eggs	12	18
Number of white eggs	4	
Number of brown eggs	8	

Activity 2

Aman, Jamuni's friend, loves chocolate! He decides to buy a bar of chocolate to share with his friends. Help him solve some problems he faced when he went to a chocolate shop.

1. A white chocolate bar contains 10- small pieces. If Aman decides to give each of his friends 2 such small pieces, how many children can share the bar ?
2. The shopkeeper sells 3 small pieces of white chocolate for Rs.4. If Aman spends Rs.40, how many such pieces of chocolate can he buy?
3. The shopkeeper charges Rs.4 for a small piece of brown chocolate. If Aman decides to buy 10 such pieces, how much money he would need?



Activity 3

Jamuni and her friends were thirsty and went to a juice shop. The juice shop had two options for orange juice: 6-litre cartons for Rs. 200 and 4-litre cartons for Rs. 150.

Which of the two cartons is cheaper?

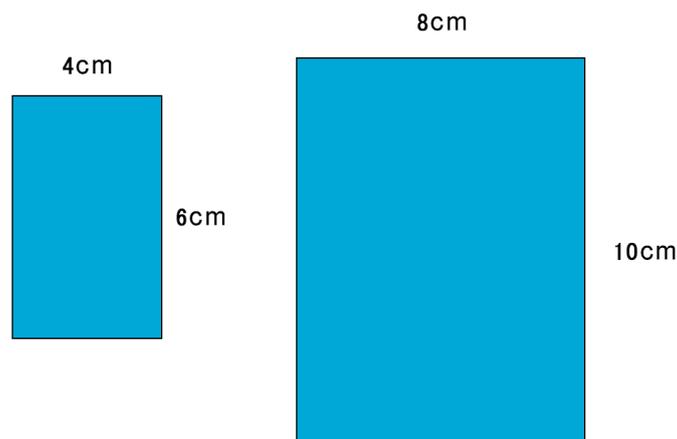
- 6-litre carton
- 4-litre carton

Tell your partner how you found the answer. Find out what method your partner used.



Activity 4

Jamuni wants to buy a square paper napkin but the shopkeeper only has rectangular ones. Look at the two rectangular paper napkins shown here. Which of them is more squarish? Why?



Lesson 2.4 : Sahir makes a poster

Please refer to this lesson on the CLIX platform

The two digital activities are based on pattern scaling and examining relationship between original and scaled shapes.

Unit 3: Ratios and Direct/Inverse Variations

Lesson 3.1 : Understanding ratio notation

Activity 1

Jamuni is sitting on the giant wheel ride. She is able to scan the entire mela scene whenever she goes to the top. She makes many observations. Can you write out Jamuni's observations in the form of a ratio?

- There is 1 boy for every 2 girls in the fair. _____
- Leena's mother is thrice as tall as her. _____
- A farmer is standing with 4 cows and 8 pigs. _____
- Geo is $2\frac{1}{2}$ times shorter than Inspector Kaata. _____

Activity 2

27 children are sitting inside a video game parlour at the mela. The ratio of girls to boys is 3:6. Which of the following statements is/are true?

- The ratio of boys to girls is 6:3. _____
- Half the children in the parlour are female. _____
- We know exactly how many boys are in the parlour. _____
- We know exactly how many girls are in the parlour. _____
- If we randomly choose 9 children in the parlour, we can expect that 3 will be girls. _____
- We can figure out how many boys there would be if the parlour was visited by 36 children. _____

Activity 3

A circus hall in the mela has 100 seats. It is divided into two zones. Zone 1 has 30 seats and zone 2 has 70 seats. A total of 80 tickets was sold for a show. All the seats in zone 1 were filled.

- What is the ratio of seats in zone 1 to seats in zone 2? _____
- What is the ratio of empty seats to occupied seats? _____
- What is the ratio of empty seats to occupied seats in zone 2? _____

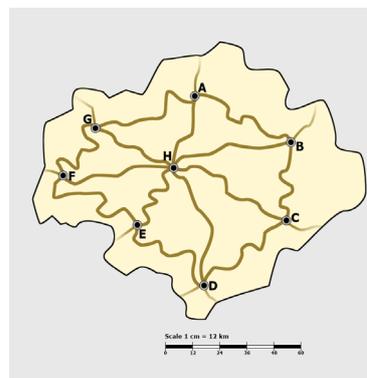
Lesson 3.2 : Map reading with Jamuni

Jamuni has an interest in map reading and a curiosity to find the distance between two places using different possible routes. She likes reading her world atlas and uses the scaling factor given in the map to calculate the exact distance between two places.

Activity 1

Carefully look at the map-scale (scale given in the map.) What do you see? Compare 1 unit of the map-scale with 1 unit of the ruler (scale) you have. Now fill the following table and put correct unit-name

	Map-scale	Real distance
1 Unit		



The ratio between map-scale and real distance can be written as $_:_$. This ratio is called the “scale factor” for a given map. It is a matter of convenience that we choose different scaling factors for showing or calculating the distance between places. Now use the above scale factor to find the distance between any two cities of your choice on the map. Think about different ways of doing this task. Fill in the following table:

(Hint: Use a thread to measure the circuitous route and using a ruler and the given map-scale, find the actual distance between these two cities)

Map-scale	Length of the thread used	Real distance
1 cm = 12 km		
1 cm = 12 km		
1 cm = 12 km		

The other way in which a map-scale is shown on a map is by expressing a unit distance and a real distance, for example, 1 cm = 15 km which indicates that 1 cm in the map distance is equivalent to 15 km in actual distance.

Map-scale (1 cm = 15 km)	Distance on the map	Real distance
	6 cm	
	10.5 cm	
	50 cm	

Activity 2

In one map, Jamuni saw the map-scale was given in ratio form. Fill in the table by calculating the real distance.

	Thread Length	Real distance
Map-scale (1:25000)	10 cm	
	18 cm	
Map-scale (1 cm = 2.5 km)	12 cm	
	21 cm	

Activity 3

In this task, thread length or the real distance between two places are given below in the table. Fill in the missing value.

Map-scale	Distance on the map	Real distance
1:1500	50 cm	
10:2000	25 cm	
1 cm = 12 km		1800 km
1:250		500 km
	10 cm	75 km
	32 cm	960 km

Activity 4

Jamuni found that the scale for the distance is represented differently in four different maps. Help her locate the map in which the distance between the cities A and B is different from all the other maps.

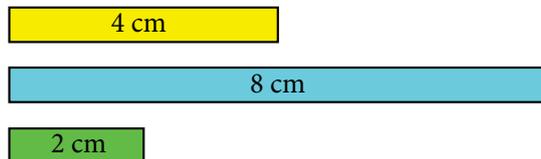
[Hint: Use a method other than cross-multiplication.]

Map	Map Distance between location A and B	Map-scale	Scaled distance between location A and B
Map 1	25 cm	1:600	
Map 2	12 cm	1:1250	
Map 3	24 cm	3:1800	
Map 4	30 cm	5:2500	

Lesson 3.3 : Finding length using strips

Activity 1

Aman, Sahir, and Leena are trying to measure the length of a sheet using strips. One of them uses only green strips; another uses only yellow strips; and the third uses blue strips.



Aman found the length as 8 units.
Leena found the length as 16 units.
Sahir found the length as 4 units.

Given these conditions, predict:
Who used the green strips ?
Who used the yellow strips ?
Who used the blue strips ?

Activity 2

Place the strips in line along the sheet's length to verify your prediction.

Now complete the following table:

Length of strip (1)	Number of strips used (n)	$1 \times n$	$1/n$

If the strip is bigger, the number of strips used is _____ (smaller/bigger).
 If the strip is smaller, the number of strips used is _____ (smaller/bigger).
 Do you see any pattern in the 1st and 2nd columns of the above table ?
 Do you see any pattern in the 3rd and 4th columns of the above table ?

We see that the ____ column contains all equal values. What does this value signify ?

Activity 3

Aman, Sahir, and Leena bought 3 sheets of different lengths from the stationery shop. They are now using the pink strip (size: 8 units) to measure its length.



Aman found the length as 24 units.
 Leena found the length as 16 units.
 Sahir found the length as 48 units.

Given these conditions, predict:
 Who had the longest sheet ?
 Who had the shortest sheet ?

Activity 4

Place the strips in line along the sheet's length to verify your prediction.

Now complete the following table:

Length of strip (1)	Number of strips used (n)	1xn	1/n

If the sheet is bigger, the number of strips used is _____ (smaller/bigger).
 If the sheet is smaller, the number of strips used is _____ (smaller/bigger).
 Do you see any pattern in the 1st and 2nd columns of the above table ?
 Do you see any pattern in the 3rd and 4th columns of the above table ?

We see the ____ column contains all equal values. What does this value signify ?

Lesson 3.4 : Ice cubes in lemonade

Please refer to this lesson on the CLIX platform

The two digital activities help in understanding inverse proportion.

Unit 4 : Applications

Lesson 4.1 : Proportions in linear equations and probability

Activity 1

Jamuni and her friends are planning to return home from the mela via train. They are checking the train time-table and found the following train:

	Station Name	Arrival time	Departure time	Distance (in km)
Train 12345	A		08:00	0
	B	12:30	13:00	225
	C	14:30	14:40	300
	D	16:40	17:00	400
	E	19:00		500

- Plot a curve by putting distance travelled on the x-axis and time taken on the y-axis. Plot different stations A, B, C, D, E on the curve.
- Find the distance that the train covers between station A and B and the time it takes. Similarly, find these values for other stations in the following table.

Stations	Distance Travelled (x)	Time taken (y)	x/y
A to B			
B to C			
C to D			
D to E			

- Find the value of x/y in each case. What are the different values that you get? Do you see a pattern? Can you give a name to x/y ? Think and discuss with your friends.
- Now write an equation that satisfies the above data table in terms of x and y .

Activity 2

Jamuni visits a stall where 3 buckets are kept. Bucket A has 2 red balls and 4 yellow balls. Bucket B has 4 red balls and 8 yellow balls. Bucket C has 7 red balls and 14 yellow balls. In order to win the prize, Jamuni has to answer the stall owner's questions correctly.

The stall owner asks: "If you pick out one ball from each bucket what is the probability that the ball will be red?" Enter your answers in the table below:

	Red balls	Yellow balls	Probability of finding a red ball
Bucket A	2	4	
Bucket B	4	8	
Bucket C	7	14	

Do you see any pattern in the last column of the table? Can you explain why you see this pattern?

Lesson 4.2 : Compound ratio and proportion

Activity 1 : Compound Proportion

1. Jamuni's parents are construction workers. She observed that a team of construction workers can construct a wall of 400 metres in 12 days by working 8 hours everyday. How long will it take if the wall size is 600 metre and the workers put in 9 hours everyday ?
2. Jamuni's mother deposited Rs. 4500 in a bank and received Rs. 360 after two years. How much interest amount will she get in 5 years if she deposited Rs. 6000?

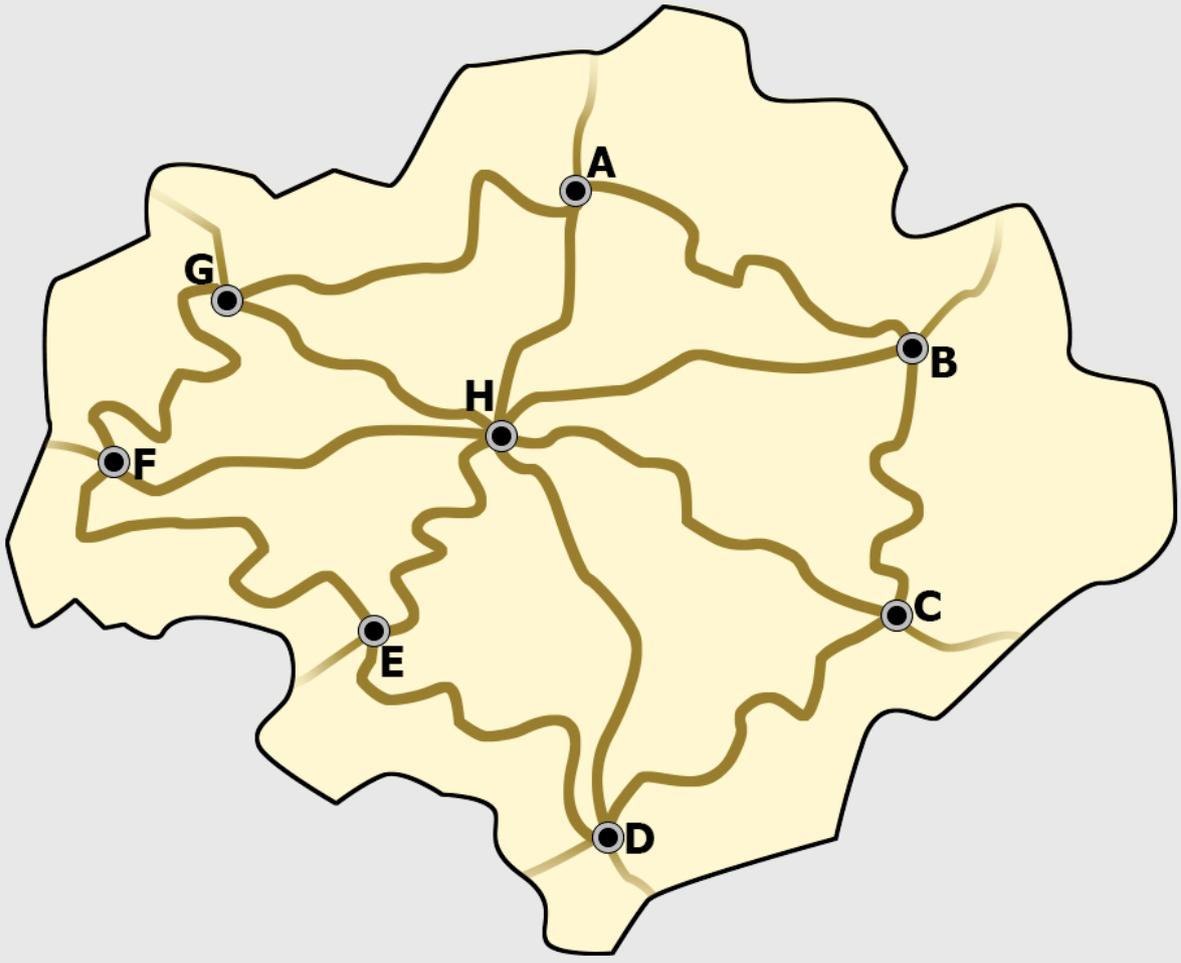
Activity 2: Mixture problem - Mini recipe problem

Jamuni, Aman, Leena, Sahir are sitting at a chai ki dukan. The recipe used for making tea for 4 persons is provided here:

- Tea powder - 2 teaspoons
- Sugar - 4 teaspoons
- Milk - 12 teaspoons
- Water - 20 teaspoons

After half an hour, Jamuni's parents also join the group, and they all decide to have a cup of tea. List the ingredients and their amounts to make tea for 6 persons now, which would taste exactly the same as the tea made earlier.

- Tea powder - ___ teaspoons
- Sugar - ___ teaspoons
- Milk - ___ teaspoons
- Water - ___ teaspoons



Scale 1 cm = 12 km





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